

For all perturbation with wave numbers less than those obtained from condition (11) the instability will develop as it drifts upwards with the phase velocity of Eq. (10). From these results and the relation between λ and k we can write the ratio of wavelengths to phase velocities for film boiling of a moving and a stationary liquid:

$$\frac{\bar{\lambda}}{\lambda} = \left(1 + \frac{3}{2}b\right) \left[\frac{3b^3 + 8b^2 + 7b + 2}{54(1+b) \left(\frac{1}{3} + \frac{1}{2}b\right)^3} \right]^{0.5}, \quad (12)$$

$$\frac{\omega/k}{\bar{\omega}/k} = 1 + b. \quad (13)$$

It can be seen from Eqs. (12) and (13) that in film boiling an increase of the liquid velocity (the parameter b) leads to an increase of the phase velocity and a decrease of the wavelength. This change of the parameters of interphase surface waves intensifies convective mixing, and, as a result, the amount of heat transmitted is increased, compared with the case of boiling of a stationary liquid.

NOTATION

x, y , longitudinal and transverse coordinates; P , pressure; ρ , density; σ , surface tension; δ , film thickness; j , transverse mass flux; W , liquid velocity; u , longitudinal vapor velocity; μ , dynamic viscosity; ν , kinematic viscosity; $g = 9.81(\rho_1/\rho_2 - 1)$, reduced acceleration due to gravity; ω , angular frequency; k , wave number; λ , wavelength; q , heat flux density, constant over the entire heater surface; c , friction coefficient; r , heat of vaporization; 1, liquid; 2, vapor; 0, stationary value; \sim , the case $W = 0$; U , mean velocity; t , time.

LITERATURE CITED

1. S. S. Kutateladze, Basic Heat Transfer Theory [in Russian], Atomizdat, Moscow (1979).

MASS-TRANSFER EFFECT IN VAPORIZATION IN VACUUM

A. V. Kondratov and A. A. Potapenko

UDC 536.422.4

The results of theoretical and experimental investigation of mass transfer in the case of vaporization in vacuum from a point source onto a substrate of arbitrary form are given.

The investigation of mass-transfer processes in vaporization in vacuum is of great theoretical and applied importance, since the thermovacuum treatment of materials is widely used in energetics, mechanical engineering, electronics, and other engineering fields.

One method of investigating mass transfer is to study the condensate profiles obtained on substrates. In connection with this, it is of interest to consider a probabilistic model of vacuum vaporization in an arbitrary plane cross section of a spraying system containing a point vaporizer and a substrate of arbitrary form.

The polar coordinate system $O\rho\phi$ is introduced in the given cross section; the equation of the substrate then takes the form

$$\rho = \Phi(\phi), \quad (1)$$

where $\Phi(\phi)$ is a differentiable function on the segment $[\phi_1, \phi_2]$. Suppose that a source vaporizing the given interval in the time interval $[0, \tau_1]$ toward the surface in Eq. (1) is placed at point 0. It is assumed that there are no processes of revaporization or migration of adatoms of the condensate the substrate. Presuming that the trajectory of any vaporizing

Agrophysical Scientific-Research Institute, Leningrad. Translated from *Inzhenerno-Fizicheski Zhurnal*, Vol. 44, No. 3, pp. 428-432, March, 1983. Original article submitted February 1, 1982.

particle is rectilinear, the orientational angle θ between the trajectory of this particle and the polar axis ρ will be regarded as positive when measured in the clockwise direction. It is assumed, further, that θ is a continuous random quantity, with a distribution density denoted by $p(\theta)$. The boundary of the coating formed on the substrate in Eq. (1) at time τ is described by an equation of the form

$$\rho = \Psi(\theta, \tau). \quad (2)$$

Choosing arbitrary permissible values θ_0 , τ_0 , sufficiently small increments $d\theta_0$ and $d\tau_0$, respectively — are given to each of them. It is natural to assume that the area of the curvilinear quadrangle [1] formed by the rays $\theta = \theta_0$, $\theta = \theta_0 + d\theta_0$ and the curves $\rho = \Psi(\theta, \tau_0)$, $\rho = \Psi(\theta, \tau_0 + d\tau_0)$ are proportional to the probability that the random quantity θ will fall in the interval $[\theta_0, \theta_0 + d\theta_0]$, i.e., that

$$\frac{1}{2} [\Psi^2(\theta_0, \tau_0) - \Psi^2(\theta_0, \tau_0 + d\tau_0)] d\theta_0 = kp(\theta_0) d\theta_0, \quad (3)$$

where k is the coefficient of proportionality. Integrating both sides of Eq. (3) over the interval of permissible angles θ , it may be shown that $k = S'(\tau_0)d\tau_0$ where $S(\tau_0)$ is the cross-sectional area of the vaporizing material at time τ_0 , while a prime denotes differentiation with respect to the time. Retaining terms of the order of $d\tau_0 d\theta_0$, in Eq. (3), the result obtained is

$$\Psi(\theta, \tau) \Psi'(\theta, \tau) = -S'(\tau) p(\theta) \quad (4)$$

after replacing θ_0 and τ_0 by θ and τ . Integration of Eq. (4), taking account of the initial condition $\Psi(\theta, 0) = \Phi(\theta)$, leads to the equation

$$\Psi(\theta, \tau) = \Phi(\theta) \sqrt{1 - \frac{2S(\tau)p(\theta)}{\Phi^2(\theta)}}. \quad (5)$$

Further consideration will be limited to the cases in which:

- 1) the maximum thickness and length of the covering does not exceed 10^{-2} cm and 50 cm, respectively, which permits the assumption that $S(\tau) \leq 5 \cdot 10^{-1}$ cm² at all the given τ ;
- 2) the distance of any point of the substrate from the vaporization source is no less than 5 cm;
- 3) the distribution density $p(\theta)$ does not exceed unity.

If the second factor on the right-hand side of Eq. (5) is expanded in binomial series in powers of the subtrahend and account is taken of the assumptions made, then, retaining only the first two terms, it is found that

$$\Psi(\theta, \tau) = \Phi(\theta) - \frac{S(\tau)p(\theta)}{\Phi(\theta)}. \quad (6)$$

The equation of the normal N to the substrate in Eq. (1) at the point A having a polar angle θ_0 in the polar coordinate system is now written

$$\rho = \frac{\Phi_0 \Phi'_0}{\Phi_0 \sin(\theta - \theta_0) + \Phi'_0 \cos(\theta - \theta_0)}. \quad (7)$$

Using Eq. (7), and taking into account that the difference $\Delta\theta$ between the polar angles of the point of interaction of N with the coating surface at time τ_0 is sufficiently small, then the following expression is accurate to quantities of order $\Delta\theta$

$$[\Phi_0 \Psi(\theta_0, \tau_0) + \Phi'_0 \Psi'_0(\theta_0, \tau_0)] \Delta\theta = [\Phi_0 - \Psi(\theta_0, \tau_0)] \Phi'_0. \quad (8)$$

If the sine theorem is used in considering triangle OAB , and terms of second order in $\Delta\theta$ are neglected, it is possible to write

$$\Psi(\theta_0, \tau_0) \Delta\theta = h \sin(\widehat{OAN}), \quad (9)$$

where h is the thickness of the condensate at point A and time τ_0 . Eliminating $\Delta\theta$ from Eqs. (8) and (9), using Eq. (6), and taking account of the expressions for the sine and cosine of the angles between the radius vector and the normal at points A and B , it is found that

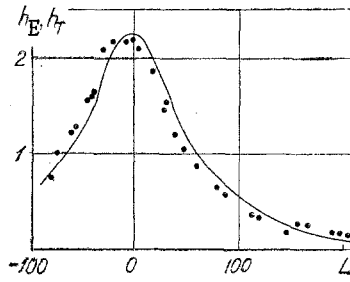


Fig. 1. Distribution of the coating thickness on the surface of a plane substrate: the curve corresponds to calculation and the points of experiment. h , μm ; L , mm .

$$h = \frac{S(\tau) p(\vartheta)}{\Phi(\vartheta)} \frac{\cos(\mathbf{OA}, \hat{N})}{\cos(\mathbf{N}, \hat{N}_1)}, \quad (10)$$

where N_1 is the normal to the surface of the coating at point B.

If the substrate is chosen in the form of the straight line

$$\rho = \frac{d}{\cos(\vartheta - \alpha)}, \quad (11)$$

where d is the distance from the point source to the substrate and α is the angle between the perpendicular dropped from the vapor source to the substrate and the polar axis, while the emission characteristic of the vapor source is described using the function

$$p(\vartheta) = \frac{m\omega\Gamma\left(\frac{m}{2}\right)}{2\sqrt{\pi}\Gamma\left(\frac{m+1}{2}\right)} \cos^m \omega\vartheta, \quad (12)$$

where m , ω are any positive numbers and Γ is a gamma function, then substitution of Eqs. (11) and (12) into Eq. (10) yields the equation of the condensate profile at the end of the spraying process

$$h(\vartheta) = \frac{m\omega S\Gamma\left(\frac{m}{2}\right)}{2d\sqrt{\pi}\Gamma\left(\frac{m+1}{2}\right)} \cos^2(\vartheta - \alpha) \cos^m \omega\vartheta, \quad (13)$$

where $S \equiv S(\tau_1)$.

To verify the theoretical results obtained, a series of sprayings has been performed in a vacuum on the order of $3 \cdot 10^{-3}$ Pa. The point vaporizer was formed in heating niobium by a 15-keV electron beam. The beam current was 120 mA.

As a result of vapor condensation on mica substrates, pellicular samples of 10×0.8 mm were obtained. The condensate thickness was determined optically and monitored by gravimetric methods. The first three sprayings were performed on annular substrate holders of radii $R_1 = 10$ cm, $R_2 = 15$ cm, and $R_3 = 20$ cm, respectively, to obtain estimates of the parameters m and ω .

From the results of measuring the condensate thickness h_{ik} ($i = -12, -11, \dots, 11, 12$; $k = 1, 2, 3$) the probability value p_{ik} is first calculated from Eq. (10), which takes the following form in the present case

$$p_{ik} = \frac{R_k}{S_k} h_{ik},$$

where $S = R_k \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} h_i(\theta) d\theta \approx \frac{\pi}{30} R_k \sum_{i=-12}^{12} h_{ik}$, and then the mean value $p_i = (1/3)(p_{i1} + p_{i2} + p_{i3})$ and

dispersion $D_i = \frac{1}{2} \sum_{k=1}^3 (p_{ik} - p_i)^2$ is found for each i . The column of values p_i is used to estimate the parameters m and ω by the nonlinear least-squares method, leading to the values:

$\omega = 0.94$, $m = 2.06$. To verify the adequacy of the mathematical model obtained on the basis of the available experimental data, the hypothesis of homogeneity of the dispersion D_i

is first tested, using the Kolkren number (significance level 0.05) $Ch_{0.05}(n, k) = D_{\max} / \sum_{i=1}^{25} D_i$,

where $D_{\max} = \max\{D_i\}$, $n = 25$ is the number of series of experiments, and $k = 3$ is the number of experiments in the series. In our case, it is found that the value of $Ch_{0.05}(25, 3)$ calculated from the experimental data is less than the corresponding critical value of the Kolkren number [2, Table 13, p. 343], and hence there is no deviation from the homogeneity hypothesis. Combining all the dispersions D_i into one weighted mean, an estimate is obtained for the dispersion associated with the experimental error, $D = 1.21 \cdot 10^{-3}$. The residual sum of squares, i.e., the sum of squares of the deviations of the experimental data from theoretical values, is $1.65 \cdot 10^{-2}$, and the corresponding residual dispersion is $7.17 \cdot 10^{-4}$. Since the dispersion ratio, i.e., the ratio of the residual dispersion to the dispersion associated with the experimental error is 0.59, and the critical value of the dispersion ratio $F_{0.05}(23, 50)$ is 1.75 [2, Table 6, p. 375], the hypothesis of adequacy of the mathematical model with respect to the experimental data may be adopted.

After determining the estimate of the vapor-source emission characteristic, niobium was sprayed from the same source, in the same conditions, onto a horizontal plane substrate holder of length 30 cm positioned a distance $d = 10$ cm from the vapor source. The experimental results are shown by the points in Fig. 1, and the results of calculation from Eq. (13) with $\omega = 0.94$, $m = 2.06$ by the curve.

The present work confirms the validity of Eq. (10), describing the dynamics of mass transfer in vacuum for neutral particles from a point source in an arbitrary plane cross section.

NOTATION

O , pole; ρ , polar radius; ϑ , polar angle; θ , orientational angle between the particle trajectory and the polar axis; τ_1 , condensation time of the coating; ϑ_1, ϑ_2 , smallest and largest values of the polar angles of substrate points; Φ_0, Φ'_0 , value of the function $\Phi(\vartheta)$ and its derivative $\Phi'(\vartheta)$ at $\vartheta = \vartheta_0$; p_{ik} , probability value at point number i for experiment number k ; S_k , cross-sectional area of the vaporizing material in experiment k ; R_k , radius of the spherical substrate holder in experiment k ; h_{ik} , coating thickness at point i for experiment k ; m, ω , parameters of the distribution density $p(\vartheta)$.

LITERATURE CITED

1. I. N. Bronshtein and K. A. Semendyaev, *Mathematics Handbook* [in Russian], GITTL, Moscow (1955).
2. V. V. Nalimov, *Using Mathematical Statistics in the Analysis of Material* [in Russian], Fizmatgiz, Moscow (1960), p. 343-375.